Radiative effects of soliton scattering from two nonlinear media

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In this paper, the problem of propagation of a self focused channel (optical beam) at an interface separating a thin film of gallium nanoparticles and monomode optical fiber has been solved. This interface is formed by two nonlinear media and has a step like inhomogeneity. The analysis has been made by applying the perturbation theory for solitons based on the inverse scattering technique. We also study the scattering of beam in the adiabatic approximation and radiative effects stipulated by the soliton scattering. A soliton solution of the nonlinear Schrödinger's equation has been obtained and the reflection and transmission coefficients of the beam (the NLSE soliton) are calculated by the Born approximation of the perturbation theory for a single nonlinear interface.

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1. Introduction

The nonlinearity created by light induced structural phase transformation in gallium has now become a subject of special interest. A large reflective nonlinearity can be achieved through optically induced phase transition in nanoscale layers of gallium nonoparticles fabricated on the tip of an optical fiber [1, 2]. The theory of reflection and transmission properties of nonlinear wave packets at an oblique angle to the interface separating two nonlinear dielectric media has been described by a number of authors [3, 4]. However, the boundary value problem at an interface with structural phase transformation induced nonlinearity of gallium like materials is a new one.

The problem of soliton propagation at such an interface has been discussed previously [5, 6] by using equivalent particle theory. But the spectral density of radiation generated during such a scattering was not analyzed, because the similar effects are beyond the equivalent particle theory of beam propagation. The purpose of this paper is to study the radiative effects accompanying the nonlinear beam scattering by an interface between a monolayer of gallium nanoparticles and monomode optical fiber. The medium inhomogeneities induced by interface are considered as perturbation in the NLSE. The analysis of nonlinear reflection coefficient of the optical beam in the region of the validity of the NLSE is the main result of this paper.

2. Gallium as a nonlinear material

For achieving optical nonlinearity in Ga-Si interface, we have discussed the properties of Ga and light-induced phase transition mechanism in Ga-Si interface. The model of such an interface is shown in Fig. 1.

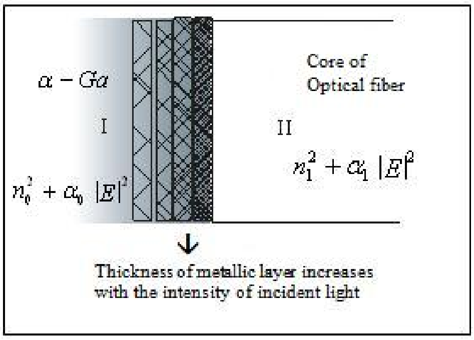

Fig. 1 Interface of thin layer of gallium nanoparticles on the core of monomde optical fiber.

Gallium has some useful properties which may be applied to the nanoscale nonlinear optics [7,8]. It has polymorphic forms called Ga-I (α–gallium) with metastable phases β, δ, ε, γ; Ga–II; Ga-III and Liquid Gallium, which have been seen in X-ray experiments. The 'metallic' behavior of Ga-II and Ga-III is similar to those of liquid gallium.

When laser beam of appropriate intensity is incident on the film of gallium nanoparticles, phase transition takes place and α-gallium in its ground state converts into its metastable state. Therefore the reflectivity or nonlinearity of the interface increases [1]. The nonlinear constitutive equation for the electric field displacement is $D = \varepsilon E + \chi^{(3)} E^3 +$ higher terms. In this equation

$$\chi^{(3)} = \frac{(\mathrm{Im}\ \varepsilon)^2 a^3 \tau}{h} \tag{1}$$

is the cubic nonlinearity [8] where a is the thickness of the metallic layer. τ is the excitation's relaxation time. It is determined by the time taken for the metastable high intensity phase to the low intensity phase [9].

$$\tau_{\text{relaxation}} = \tau_{\text{thermal}} + \tau_{\text{recrystallization}}$$

τ_{thermal} is the time for the particle to return to its initial phase, it is of order of picoseconds and $\tau_{\text{recrystallization}}$ depends upon the overcooling of phase, is of order of microsecond. So this time is in the picoseconds – microsecond range. In Eq. (1), $(\text{Im}\ \varepsilon)$ is the imaginary part of dielectric constant of metallic phase $(\varepsilon = -115.3 - i98.4)$ [10]. As gallium is a Kerr-like dielectric media, So, Let the refractive index of gallium is given by

$$n^2 = n_0^2 + \alpha_0 |E|^2 \qquad (2)$$

Where n_0 is linear part (intensity independent part) and α_0 is nonlinear part of refractive index given by

$$\alpha_0 = \frac{3\chi_0^{(3)}}{2c\varepsilon_{eff}\, n_0} \qquad (3)$$

Where c is velocity of light and ε_{eff} is the effective dielectric constant which is calculated by Maxwell – Garnet effective medium theory [10] as

$$\varepsilon_{eff} = \varepsilon_{ext} \left\{ q \frac{\varepsilon_{\text{int}} - \varepsilon_{ext}}{\varepsilon_{eff} + A(\varepsilon_{\text{int}} - \varepsilon_{ext})} + 1 \right\} \qquad (4)$$

Where ε_{ext} =1 (dielectric constant of surrounding), ε_{int} = – 115.3 – i 98.4 (dielectric constant of liquid phase of gallium nanoparticles),q = volume filling factor of the film. For spherical nanoparticles, is found to be

$$\varepsilon_{eff} = -\left(\frac{14.119 + 116.3\,A}{1 - 116.3\ A} \right)$$

Now Eq. (3) becomes

$$\alpha_0 = \frac{3\chi_0^{(3)}(116.3A - 1)}{2cn_0(14.119 + 116.3A)} \qquad (5)$$

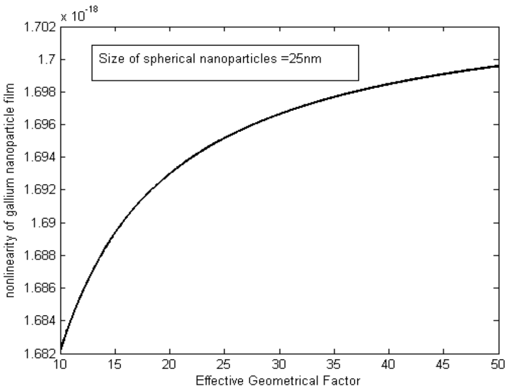

Fig. 2. Gallium's nonlinearity as a function of effective geometrical factor.

Its variation with effective geometric factor A is shown in Fig.2, which shows that the nonlinearity of gallium nanoparticles increases with effective geometrical factor and attains a constant value after a certain range of shell thickness for a constant value of size of nanoparticles.

The soliton propagation characteristics in a monomode optical fiber are also functions of interface nonlinearity which may depend upon light induced structural phase transformation effects, as we have considered here for a layer of gallium nanoparticles. In the next section a formulation of the theory of soliton propagation from gallium nanoparticle film at monomode optical fiber has been presented.

3. Formulation of the problem

Let us consider the propagation of a collimated beam of light at the adjoining nonlinear dielectric media. One is a thin film of gallium nanoparticles and other is a monomode optical fiber. Two optical media differ by refractive indices and depend on the electric field E as

$$n^2\left(x, |E|^2\right) = \begin{cases} n_0^2 + \alpha_0 |E|^2, & x < 0 \text{ in Ga} \\ n_1^2 + \alpha_1 |E|^2, & x > 0 \text{ in Optical Fiber} \end{cases} \qquad (6)$$

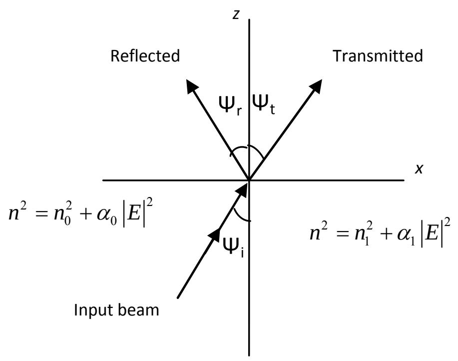

Fig.3 Interface configuration for an interface separating thin film of gallium nanoparticles and monomode optical fiber.

The propagation of the transverse electric wave is described by the wave equation

$$i\ \frac{\partial \psi}{\partial t} + \frac{\partial^2 \psi}{\partial x^2} + 2|\psi|^2\ \psi = U(x, |\psi|^2)\psi \qquad (7)$$

Two equations for each medium can be considered as a single one but with the step like potential

$$U\left(x, |\psi|^2\right) = \begin{cases} 0, & x < 0 \\ \Delta - B|\psi|^2, & x > 0 \end{cases} \qquad (8)$$

where $B = 2(\alpha_1 - \alpha_0)/\alpha_0$, $\Delta = n_0^2 - n_1^2$. In the first medium (gallium nanoparticle film) $U=0$. So an exact solution of Eq.(7) is a NLSE soliton

$$\psi_s(x,t) = 2\eta \sec h[2\eta\{x - x_0\}]. \\ .\exp[-2i\xi x + i\delta(t)] \tag{9}$$

Where 2η and 4ξ are its amplitude and velocity respectively, $\delta(t) = 4(\xi^2 - \eta^2)t$ is the phase and $x_0(t) = -4\xi t$ is the coordinate.

The velocity of the soliton envelope $(2\xi/\beta)$ is proportional to the sine of the angle of incidence ψ_i so velocity of soliton is $v = 4\xi = 2\beta\sin\psi_i$ and its amplitude (2η) is proportional to its power,

$$P = \int_{-\infty}^{\infty} |E|^2 dx = \frac{2}{\alpha_0}\int_{-\infty}^{\infty} |\psi|^2 dx = 2(4\eta)/(\alpha_0 k_0) \tag{10}$$

Where 4η is the value of the integral motion of the NLSE, which has a meaning of a number of photons bound in the soliton as a bound state. Its amplitude is given by $2\eta = \alpha_0 P/4 = \sqrt{\beta^2 - n_0^2}$

The reflection coefficient as a ratio of the incident beam power P_i to the power of the reflected beam P_r

$$R = (P_r/P_i) = (N_r/N_i) \tag{11}$$

And the transmission coefficient T is

$$T = 1 - R \tag{12}$$

4. Formalism of the soliton perturbation theory

The slow change of soliton parameters can be described in the framework of so called adiabatic approximation, assuming that the beam shape is still defined by the expression (9) but its parameters are changed. The corresponding equations for the NLSE soliton parameters can be expressed as

$$i\frac{\partial\psi}{\partial t} + \frac{\partial^2\psi}{\partial x^2} + 2|\psi|^2\psi = \varepsilon R(\psi) \tag{13}$$

Here ε <<1. Now we use the soliton perturbation theory based on the inverse scattering technique [11] to calculate the adiabatic shape of the soliton solution. The reflection coefficient of the nonlinear light beam from the interface is calculated and for (i) b<<1 it has form

$$R = R_0\left[(1+b^2) - \frac{16}{3}\frac{B\xi^2 b^2}{\Delta}\left(1 + \frac{2b^2}{15}\right) + \frac{B^2\xi^4 b^4}{\Delta^2}\left(\frac{384}{5} + \frac{2149}{4}b^2\right)\right] \tag{14}$$

Where $b = \eta/\xi$ and $R_0 = \frac{\Delta^2}{2^8\xi^4}$. It corresponds to the so-called 'light soliton' or a small power beam and for (ii) b>>1, the case of a 'heavy soliton' (large power beam) is characterized by exponentially small emission due to a large bound energy of soliton

$$R = \frac{\pi\sqrt{2}}{2^9\xi^4\sqrt{b}} e^{-\frac{\pi b}{2}}\left[\Delta - \frac{B}{12}b^4\xi^2\right]^2 \tag{15}$$

Fig.4(a) shows variation of reflection coefficient with parameter $(b = \eta/\xi)$ at a constant power level. From this curve we observe that as the angle of incidence increases, the reflection coefficient decreases upto a very small extent and Fig.4(b) shows that on keeping angle of incidence constant, the peak of reflection coefficient shifts toward smaller values of b with increase in power of incident soliton. So we can obtain high reflection for large power beams. Hence high reflection is achieved for high power beam, incident on the interface at smaller angles; these results are experimentally justified [12]. Here n_1=2.93, n_0=1.5, β=4.3, $B=2(\alpha_1-\alpha_0)/\alpha_0$=0.26, $\gamma = \Delta/2\eta^2 B = 2.22$ and P is the power of incident soliton beam.

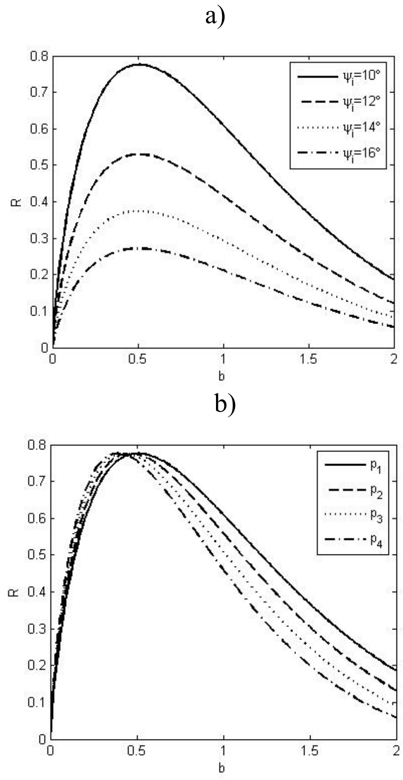

Fig.4 Dependence of beam reflection coefficient on the parameter $(b = \eta/\xi)$ (a) for various angle of incidence and (b) for various power levels with $P_1 < P_2 < P_3 < P_4$.

5. Beam scattering by an interface

Here the incident soliton envelope is treated as a particle whose position is given by $x_0(t)$. The change in refractive index at the interface gives rise to a perturbative potential $U(x_0)$ shown by Eq.(8). The adiabatic approximation gives us an equivalent particle motion in the external potential [13]. $V(r)=\Delta(1+\tanh r)-2\eta^2 B\left(0.66+\tanh r-(\tanh^3 r)/3\right)$

The shape of potential is described in Fig.5. This potential shows step like inhomogeneity of two nonlinear media.

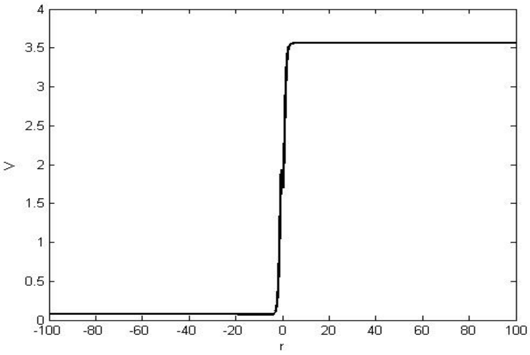

Fig.5 Shape of effective potential for the interface between gallium nanoparticles and monomode optical fiber.

6. Results and discussion

From Fig.5, we observe that

6 a. Maximum and minimum points are obtained at $\tanh r_0 = \pm\sqrt{1-\gamma}$. Their existence means the possibility of the capture of a soliton by the interface i.e. the transformation of an input self-focused channel into a trapped stationary nonlinear surface wave. Since these points are obtained at the interface, it represents the trapped NSW slightly displaced from its stable equilibrium position undergoing oscillations in the vicinity of the interface.

6 b. The dependence of the refractive angle (ψ_t) on the incident angle (ψ_i) is determined by an expression

$$\sin^2\psi_t = \sin^2\psi_i - \frac{\Delta}{\beta^2} + \frac{B\alpha_0^2 P^2}{48} \qquad (16)$$

For $\sin^2\psi_i > (\Delta/\beta^2)$ *or* $\sin\psi_i > \sqrt{\Delta}/\beta$

$$\Rightarrow \psi_i > \sin^{-1}\left[\sqrt{4.678}/4.3\right] \Rightarrow \psi_i > 30.37^0$$

The beam is always transmitted by the effective potential which is shown by Fig. 6.

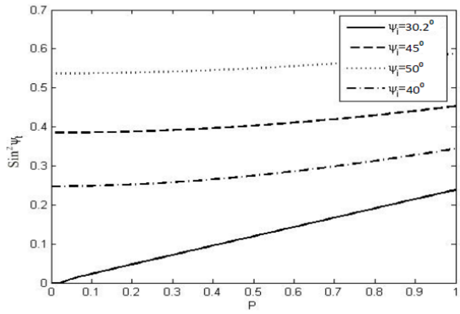

Fig. 6. Variation of sin² ψ_t as a function of its input power for various values of input beam angles.

This curve shows that all the beams which are incident at an angle greater than (30.37°) will be transmitted by the interface. It is also observed that for a constant angle of incident the angle of refraction increases with increase in intensity (power) of incident beam.

6 c. For angles smaller than (30.37°) there is a threshold power, above which beam is transmitted to the second medium (optical fiber). This threshold power is determined either by the condition

$$8\xi_0^2 = V(+\infty) = 2\Delta[1-(2/3\gamma)]$$

$$\text{for } (\Delta/36) < \xi_0^2 < (\Delta/4)$$

i.e. the result is $\eta_{thr}^2 = 0.75(\Delta - 4\xi_0^2)/B$, in a range ($11.40^0 < \psi_i < 30.37^0$) or by the condition

$$8\xi_0^2 = V_{\max} \quad \textit{for } \xi_0^2 < \frac{\Delta}{36} \textit{ or } (\psi_i < 11.40^0)$$

In this range graph is plotted between $\sin^2\psi_t$ and η (power of incident beam) in Fig.7, which shows existence of threshold power.

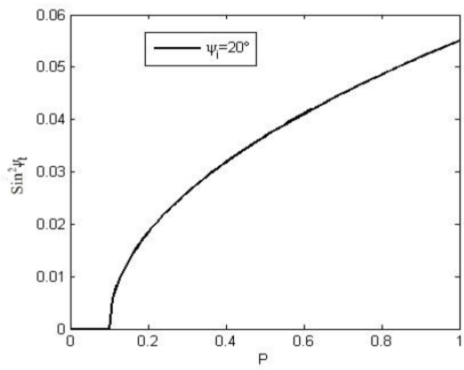

Fig.7. Variation of sin² ψ_t as a function of its input power for **(Δ / 36) < ξ_1^2 < (Δ / 4)**

The fact that the transmitted beam angle is equal to zero for $\xi < \xi_{thr}$ (or $\eta < \eta_{thr}$) does not mean that there is no field in the right medium at all. In the adiabatic theory we can predict only the propagation of the main self-focused channel -a beam of the soliton like form. When this beam is reflected by the interface, a portion of its energy is transmitted by it, whose power turns out to be a small quantity. These effects are described by radiative effects.

7. Conclusions

In conclusion,, by means of the perturbation theory for solitons we have investigated the scattering of optical beams by nonlinear interface between gallium nanoparticles and monomode optical fiber. It is found that if angle of incident is greater than 30.37^0 the beam is always transmitted and for smaller angles there is a threshold power above which beam is transmited in the fiber medium. The solution of NLSE in the form of the nonlinear self-focused channel is a soliton solution and the interface is considered as a perturbation. In the framework of the Born approximation of the soliton perturbation theory we have calculated the soliton reflection coefficient which is a function of beam power and angle of incidence of optical beam on the interface. It is found that for small values of $(b = \eta / \xi)$ reflection coefficient increases with increase in intensity and decreases with increase in angle of incidence of soliton. This result is experientially proved [12]. It decreases exponentially with growth of intensity for b>>1. These results are useful for building of all-optical integrated devices using interfaces, e.g, optical limiters, bi-stable switchers, upper and lower threshold devices, etc.

Reference

[1] K.F. MacDonald, V.A. Fedotov, N.I. Zheludev, Applied Physics Letters. **82**, 1087–1089 (2003).
[2] S. Pochan, K. F. MacDonald, R. J. Knize, N. I. Zheludev, Phy. Rew. Lett., **92**, 145702.1 (2004).
[3] L. A. Nesterov, Opt. Spektrosk. **64**, 1166 (1988).
[4] A. V. Aceves, J. V. Moloney, A. C. Newell, Phys. Rev. A, **39**, 1809-1827 (1989).
[5] P. Naruka, S. Bissa, A.K. Nagar, A. Bhargava, Soliton propagation through an interface between gallium nanoparticle film and monomode optical fiber. Presented at the 52nd DAE Solid State Physics symposium, BARC-BRNS, Mysore, India (December 27-31, 2007), pp. 623-624.
[6] Preeti Naruka, Shivangi Bissa,A. K. Nagar, A. Bhargava, Study of Soliton propagation at an interface separating two nonlinear media. Presented at the 53rd DAE Solid State Physics symposium, BARC-BRNS, Mumbai, India. (December 16-20, 2008), pp. 773-774.
[7] Ajoy Ghatak, K. Thyagrajan, Introduction to Fiber Optics, Cambridge University Press, U.K. (2004), pp. 341-344.
[8] N. I. Zheludev, .Nonlinear optics on Nanoscale. Contempory Phy., **43**, 365-377 (2002).
[9] N. I. Zheludev,. J.Opt.A: Pure Appl.Opt., **8**, S1-S8 (2006).
[10] V. A. Fedotov, V. I. Emil'yanov, K. F. MacDonald, N. I. Zheludev,. J.Opt.A:pure appl.Opt., **6**, 155-160 (2004).
[11] Yu. S. Kivshar, A. M. Kosevich, O. A. Chubykalo, Phy. Review A., **41**, 1677-1688 (1990).
[12] Ladislav Jankovic, Hongki Kim, George Stegeman, Silvia Carrasco, Lluis Torner, Mordechai Katz, Optics Lett., **28**, 2103-2105 (2003).
[13] Yu. S. Kivshar, A. M. Kosevich, O. A. Chubykalo, Zh. Eksp. Teor. Fiz., **93**, 968-977 (1987).

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