Classical effects in the above threshold ionization and high harmonic generation

A. POPA
National Institute for Laser, Plasma and Radiation Physics, Laser Department, P.O. Box MG-36, Bucharest, Romania 077125

We consider a system composed of an atom under the action of a very intense laser field. We consider the Corkum's hypothesis, that the wave packet which is emitted by tunneling in the photo-ionization process can be approximated by a classical electron. Using this hypothesis, we prove that, at the end of the laser pulse, the ionized electron is situated at very high distance from nucleus and its energy is equal to the drift energy after ionization, in agreement with the above threshold ionization theories for nonrelativistic interaction. We prove that the energy of the ionized electron contains a supplementary term in the relativistic case, which leads to a different type of above threshold ionization spectrum from the nonrelativistic case. Our approach predicts the suppression of the harmonics in the strong relativistic regime, in the frame of Corkum's model of high harmonics generation.

(Received May 20, 2011; accepted June 3, 2011)

Keywords: Laser, Threshold ionization spectrum,. Harmonic generation, Corkum's model

1. Introduction

The development of very intense laser beams, corresponding to electrical fields of the order of an atomic unit, led to a new phenomenon, above threshold ionization (ATI), which has been evidenced by Agostini et al [1]. This is a quantum phenomenon, by which the electromagnetic field of the laser beam transfers an energy to the electron, equal to an integer multiple of the field quantum, which is sufficiently high for the electron to leave the atom. The existence of the threshold at which the ATI phenomenon is possible leads to a sequential character of the electron motion, in which three phases can be distinguished. The first phase takes place in the atomic core domain where the electric potential of the nucleus cannot be neglected, whose dimension is of the order of a few Bohr radia; it consists of the multiphoton absorbtion, after which the electron leaves the atom. The second phase takes place in a very large ionization domain, whose dimension is of the order of thoundsands of Bohr radia, where the electrostatic potential of the nucleus is negligible and the electron is free. In the third phase, after oscillating in the electromagnetic field in the ionization domain, the electron returns in the atomic domain and transfers its energy to the field [high harmonic generation (HHG) phenomenon], or rescatters in the field of the nucleus.

A large number of semiclassical models lead to results in very good agreement with the experiments. We recall the classical model of Corkum [2], which leads to the most precise--to our knowledge--expression from literature for the cutoff of the high harmonic radiation, the model of the rescattering of the electrons of Paulus et al [3], which leads to an accurate expression of the maximum value of the electron energy in the ATI spectrum, or the analysis of the classical effects in ATI and HHG phenomena presented by Becker et al [4]. Classical orbits were able to explain multiple features of HHG and ATI spectra in the papers of Salieres et al [5], Bellini et al [6] and Kopold et al [7]. Generally, these semiclassical approaches are based on Feynman's path integral theory [8], at the classical limit, when the number of the quantum trajectories which are taken into account are very few.

The above results complement the quantum treatments for the domain where the electrostatic interaction between electron and nucleus is taken into account, as those of Reiss [9], Faisal and Radozycki [10], [11], Levenstein et al [12] and Chirila et al [13].

The paper is structured as follows. In Section 2 we present the initial hypotheses and the equations of interaction between electromagnetic field and electron in the ionization domain. In Section 3 we use again the relations from Section 2, to calculate the average distance between electron and nucleus and the kinetic energy of the electron at the end of the laser pulse, necessary in the analysis of the ATI phenomenon and to study the effect of the HHG suppression in the strong relativistic regime, in the frame of the Corkum's model.

The equations are written in the International System.

2. Basic theory

2.1 Initial hypotheses

We analyze a system composed of an atom interacting with a very intense electromagnetic elliptically polarized field. We consider the following initial hypotheses:

(h1) The electromagnetic field is of the type produced by a very intense laser beam, and the value of the intensity of its electric field is of the order of one atomic unit, namely $5.1423\times10^{11} V/m$, or greater. In this case the electric field of the laser becomes comparable to the electric field of the nucleus, so the electron is free to escape from the atomic potential [14].

(h2) We assume that the hypothesis of the Corkum's model presented in Ref. [2] is valid, namely that the wave packet which is emitted by tunneling in the photo-ionization process can be approximated by a classical electron. This hypothesis is supported by the fact that the probability of atom ionization under the action of a very intense electromagnetic field is maximum near each peak of the laser electric field, and the emission of the wave packet takes place in a very short time. On the other hand, in virtue of the Simpleman's theory [15], the velocity of the electron is negligible after tunneling, which means that the length of the wave packet is very small. The Corkum's hypothesis is confirmed by the fact that the classical trajectories which are connected to the properties of the quantum equation, lead to accurate models, as those presented in the papers [2]-[7].

(h3) We suppose that one electron is situated in the ionization domain, where the electrostatic interaction with nucleus is neglected.

(h4) The electromagnetic field is elliptically polarized. In a Cartesian system of coordinates, the intensity of the electric field and of the magnetic induction vector, denoted respectively by $\overline{E}$ and $\overline{B}$, are polarized in the plane xy, while the wave vector, denoted by $\overline{k}$, is parallel to the axis z. The expression of the electric field is

$$\overline{E} = \bar{i}E_1 \cos\eta + \bar{j}E_2 \sin\eta \qquad (1)$$

with

$$\eta = \omega t - kz + \varphi \qquad (2)$$

where $\bar{i}$ and $\bar{j}$ are the versors of the x and y axes, E_1, E_2 are the amplitudes of the oscillations of the electric field in the x and y directions, ω is the angular frequency of the electromagnetic field and φ is an arbitrary phase.

From the properties of the electromagnetic field, it follows that the corresponding magnetic induction vector is

$$\overline{B} = -\bar{i}B_2 \sin\eta + \bar{j}B_1 \cos\eta \qquad (3)$$

with

$$E_1 = cB_1 \quad \text{and} \quad E_2 = cB_2 \qquad (4)$$

where B_1 and B_2 are the amplitudes of the oscillations of the magnetic field in the y and x directions and c is the light velocity.

From equation of the electric field $\overline{E} = -\partial\overline{A}/\partial t$, it follows that the expression of the magnetic vector is

$$\overline{A} = -\bar{i}A_1 \sin\eta + \bar{j}A_2 \cos\eta \qquad (5)$$

where

$$E_1 = \omega A_1 \quad \text{and} \quad E_2 = \omega A_2 \qquad (6)$$

(h4) In virtue of numerous approaches from the literature [4], [5], we suppose that the Simpleman's theory [15] is valid, and, after tunnelling, the electron enter in the ionization domain with negligible velocity. The following relations are valid at the beginning of the ionization domain:

$$t = t_0, \quad x = y = z = 0, \quad v_x = v_y = v_z = 0, \quad \eta = \eta_0$$

and

$$\overline{A} = \overline{A}_0 \qquad (7)$$

where v_x, v_y and v_z are the components of the velocity of the electron.

2.2 Classical analysis of the ionization domain

Taking into account (1) and (3), the classical equations of motion of the electron are

$$m\frac{d}{dt}(\gamma v_x) = -eE_1 \cos\eta + ev_z B_1 \cos\eta \qquad (8)$$

$$m\frac{d}{dt}(\gamma v_y) = -eE_2 \sin\eta + ev_z B_2 \sin\eta \qquad (9)$$

$$m\frac{d}{dt}(\gamma v_z) = -ev_x B_1 \cos\eta - ev_y B_2 \sin\eta \qquad (10)$$

where m is the electron mass, e is the absolute value of the electron charge,

$$\gamma = \left(1 - \beta_x^2 - \beta_y^2 - \beta_z^2\right)^{\frac{1}{2}} \qquad (11)$$

and $\beta_x = v_x/c$, $\beta_y = v_y/c$ and $\beta_z = v_z/c$.

Taking into account (4), the equations of motion become

$$\frac{d}{dt}(\gamma\beta_x) = -a_1\omega(1-\beta_z)\cos\eta \qquad (12)$$

$$\frac{d}{dt}(\gamma\beta_y) = -a_2\omega(1-\beta_z)\sin\eta \qquad (13)$$

$$\frac{d}{dt}(\gamma\beta_z) = -\omega(a_1\beta_x \cos\eta + a_2\beta_y \sin\eta) \qquad (14)$$

where

$$a_1 = \frac{eE_1}{mc\omega} \quad \text{and} \quad a_2 = \frac{eE_2}{mc\omega} \qquad (15)$$

are relativistic parameters.

The solution of a system similar to (12)-(14), in the case of the electron motion in a linearly polarized field, is given in [16], [17]. In this section we will present briefly the solution in the most general case, that of an elliptically polarized field, in order to calculate β_x, β_y and β_z, which are necessary in our analysis.

We multiply (12), (13) and (14}), respectively, by β_x, β_y and β_z. Taking into account that $\beta_x^2+\beta_y^2+\beta_z^2=1-1/\gamma^2$, their sum leads to

$$\frac{d\gamma}{dt}=-\omega\left(a_1\beta_x\cos\eta+a_2\beta_y\sin\eta\right) \quad (16)$$

From (14) and (16) we have $d(\gamma\beta_z)/dt=d\gamma/dt$. We integrate this relation, taking into account the initial conditions (7) and the relation (2), and obtain

$$\frac{1}{\gamma}=1-\beta_z=\frac{1}{\omega}\frac{d\eta}{dt} \quad (17)$$

We integrate (12), taking into account (17) and the initial conditions (7) and obtain

$$\beta_x=-\frac{a_1}{\gamma}\left(\sin\eta-\sin\eta_0\right) \quad (18)$$

Similarly, integrating (13) and taking into account (7) and (17), we obtain

$$\beta_y=-\frac{a_2}{\gamma}\left(\cos\eta_0-\cos\eta\right) \quad (19)$$

We substitute the expressions of β_x, β_y and β_z, respectively, from (18), (19) and (17) into (11) and obtain the expression of γ:

$$\gamma=1+\frac{a_1^2}{2}\left(\sin\eta-\sin\eta_0\right)^2+\frac{a_2^2}{2}\left(\cos\eta_0-\cos\eta\right)^2 \quad (20)$$

From (17) and (20) obtain

$$\beta_z=1+\frac{a_1^2}{2\gamma}\left(\sin\eta-\sin\eta_0\right)^2+\frac{a_2^2}{2\gamma}\left(\cos\eta_0-\cos\eta\right)^2 \quad (21)$$

We observe that the above relations for β_x, β_y and β_z and γ can be written for the motion of the electron in a circularly polarized field, substituting $E_1=E_2$, $B_1=B_2$ and $a_1=a_2$. Also, these relations can be written in the case of a linear polarized field, when $E_2=0$, $B_2=0$, $a_2=0$.

3. Classical effects in ATI and HHG phenomena

3.1 Analysis of the ATI phenomenon

In order to analyze the ATI phenomenon in the light of the relations deduced in the previous section, we evaluate the kinetic energy of the electron and the average distance between the electron and nucleus at the end of the laser pulse.

The kinetic energy of the electron, denoted by E_{kin}, at an arbitrary time moment, is given by {see for example equation (16.7) from [16]}

$$\left(E_{kin}+mc^2\right)^2=m^2c^4+c^2p^2 \quad (22)$$

which yields

$$E_{kin}=-mc^2+mc^2\sqrt{1+\frac{p^2}{m^2c^2}}=\frac{1}{2}\frac{p^2}{m}-\frac{1}{8}\frac{p^4}{m^3c^2}+\frac{1}{16}\frac{p^6}{m^5c^4}-.... \quad (23)$$

From (18), (19) and (21) we obtain the kinetic momentum of the electron, which, in virtue of (5), (6) and (15) can be written

$$\bar{p}=-mca_1\left(-\frac{\bar{A}\cdot\bar{i}}{A_1}+\frac{\bar{A}_0\cdot\bar{i}}{A_1}\right)\bar{i}+-mca_2\left(-\frac{\bar{A}\cdot\bar{j}}{A_2}+\frac{\bar{A}_0\cdot\bar{j}}{A_2}\right)\bar{j}+ \quad (24)$$
$$\gamma mc\beta_z\bar{k}=e\bar{A}-e\bar{A}_0+\gamma mc\beta_z\bar{k}$$

Similarly, the component parallel to the direction of the propagation of the wave becomes:

$$\gamma mc\beta_z\bar{k}=\frac{mca_1^2}{2}\left(-\frac{\bar{A}\cdot\bar{i}}{A_1}+\frac{\bar{A}_0\cdot\bar{i}}{A_1}\right)^2\bar{k}+\frac{mca_2^2}{2}\left(-\frac{\bar{A}\cdot\bar{j}}{A_2}+\frac{\bar{A}_0\cdot\bar{j}}{A_2}\right)^2\bar{k}= \quad (25)$$
$$\frac{e^2}{2mc}\left(A^2+A_0^2\right)\bar{k}-\frac{e^2}{mc}\left[\left(\bar{A}\cdot\bar{i}\right)\left(\bar{A}_0\cdot\bar{i}\right)+\left(\bar{A}\cdot\bar{j}\right)\left(\bar{A}_0\cdot\bar{j}\right)\right]\bar{k}$$

The relations (24) and (25) give the variation of $\bar{p}$ with $\bar{A}$. At the end of the laser pulse, when $\bar{A}$ decreases to zero in a very short time, the expression of the kinetic momentum becomes:

$$\bar{p}=-e\bar{A}_0+\frac{e^2A_0^2}{2mc}\bar{k} \quad (26)$$

Replacing this relation in (23), it follows that the expression of the kinetic energy of the electron, at the end of the laser pulse, becomes

$$E_{kin}=\frac{1}{2m}\left[\left(e\bar{A}_0\right)^2+\frac{\left(e\bar{A}_0\right)^4}{(2mc)^2}\right]-\frac{1}{8m^3c^2}\left[\left(e\bar{A}_0\right)^2+\frac{\left(e\bar{A}_0\right)^4}{(2mc)^2}\right]+... \quad (27)$$

In the nonrelativistic regime, only the term $\left(e\overline{A}_0\right)^2/2m$ is significant, all the other terms being negligible. We obtain the well known formula for the kinetic energy which leads to the ATI spectrum, because it is proved that $e\overline{A}_0$ and $\left(e\overline{A}_0\right)^2/(2m)$ are, respectively, the drift momentum and the drift energy measured at the detector (see, for example, Ref. [4], pag. 41). Quantum evaluations [10], [11] show that the drift energy is equal to an integer multiple of the field quantum energy.

In the relativistic regime, our relations show that the field transfers continuously to the electron a small energy, which leads to a supplementary component of the kinetic energy, due to the motion in the z direction. Since this energy is not equal to an integer multiple of the quantum energy of the field, it follows that the ATI spectrum is different in the relativistic case, compared to the spectrum in the nonrelativistic case. This is a qualitative classical explanation of the difference between the ATI spectra in the relativistic and nonrelativistic cases, but the exact calculation of the ATI spectrum in the relativistic regime results from quantum calculations [10], [11].

We evaluate now the average distance between the electron and nucleus at the end of the laser pulse. For simplicity, we limit the analysis to the case when the field is linearly polarized and $a_2 = 0$. We calculate the distance between the electron and the atomic core domain, namely the value of the x coordinate at an arbitrary time moment, corresponding to an arbitrary phase η. We integrate (18), taking into account the initial conditions (7) and the relation (17) and obtain

$$x = -ca_1\int_{t_0}^{t}\frac{1}{\gamma}(\sin\eta - \sin\eta_0)dt = -\frac{ca_1}{\omega}\int_{\eta_0}^{\eta}(\sin\eta - \sin\eta_0)d\eta = \frac{ca_1}{\omega}\left[\cos\eta - \cos\eta_0 + \sin\eta_0(\eta - \eta_0)\right] \quad (28)$$

The maximum distance from the nucleus occurs for $\beta_x = 0$, which, in virtue of relation (18), corresponds to the phase $\eta_M = \pi - \eta_0$. For this value, in virtue of (28), we obtain the expression of the maximum value of x, denoted by x_M. This is a function of η_0, given by the following relation

$$x_M = -\frac{ca_1}{\omega}\left[2\cos\eta_0 - \sin\eta_0(\pi - 2\eta_0)\right] \quad (29)$$

For $\eta_0 = 0$, $\pi/2$, π and $3\pi/2$, we have, respectively, $x_M = -2ca_1/\omega$, 0。 $2ca_1/\omega$, and 0. On the other hand, in virtue of the tunelling model [2], [14] the ionization rate is maximum when the electric field is maximum, or the value of A_0 is minimum. This corresponds to the values $\eta_0 = 0$ and $\eta_0 = \pi$. Consequently the most probable value of the maximum distance from nucleus, which corresponds to the maximum ionization rate, namely to $\eta_0 = 0$ and $\eta_0 = \pi$, is $|x_M| = 2ca_1/\omega$. It follows that the order of magnitude of the average distance between electron and nucleus, at the end of the laser pulse, is roughly equal to $|x_M|/2 = ca_1/\omega$. A simple calculation shows that this distance is very big compared to a_0, the first Bohr radius. It is of the order $1000a_0$ for a beam intensity $I = 10^{21}\,W/m^2$, and $\omega = 1.777\cdot 10^{15}\,rad/s$, in the case of a Nd:YAG laser.

In virtue of the above analysis, it follows that at the end of the laser pulse the electrons are situated at very long distances from nucleus, and the time of flight detector can easily measure their kinetic energies.

3.2 Analysis of the HHG phenomenon

Two arguments for Corkum's model are the very accurate prediction of the expression for the cutoff of the high harmonic radiation, and the fact that the HHG phenomenon can not take place in the case of the elliptic or circular polarizations of the electromagnetic field [18], because, in this case, the electron cannot return back to the atomic core domain. In this section we will study a third argument, namely the suppression of the high harmonics, due to the displacement of the electron in the z direction. We show that the suppression is greater for the low-frequency part of the HHG spectrum than for the high-frequency part. In this respect, our relations lead to theoretical results in agreement with the data presented by [19], [20].

Since the electron comes back to the atomic core domain only when the laser field is linearly polarized, we limit the analysis to this case, when $E_2 = 0$, $B_2 = 0$, $a_2 = 0$ and the motion of the electron takes place in the xz plane. In virtue of the relation (28), the electron returns to the atomic core when $x = 0$ and the phase of the field is equal to η_1, as results from the following equation

$$\cos\eta_1 - \cos\eta_0 + \sin\eta_0(\eta_1 - \eta_0) = 0 \quad (30)$$

which has also been deduced in the frame of Corkum's model [2], [14].

The integration of equation (21) between time $t = 0$ and the time t_1 corresponding to the phase η_1, leads to the distance z_1, when the electron returns toward the atomic core. With the aid of the relation (17), we obtain the value of z_1 normalized to a_0, as follows

$$\frac{z_1}{a_0} = \frac{ca_1^2}{2a_0}\int_{t_0}^{t_1}\frac{1}{\gamma}(\sin\eta - \sin\eta_0)^2 dt = \frac{ca_1^2}{2\omega a_0}\int_{\eta_0}^{\eta_1}(\sin\eta - \sin\eta_0)^2 d\eta = \frac{ca_1^2}{2\omega a_0}g(\eta_1,\eta_0) \quad (31)$$

where

$$g(\eta_1,\eta_0)=2\sin\eta_0(\cos\eta_1-\cos\eta_0)-\frac{1}{4}(\sin 2\eta_1-\sin 2\eta_0)+(\eta-\eta_0)\left(\frac{1}{2}+\sin^2\eta_0\right) \quad (32)$$

The quantities η_0, η_1 and $g(\eta_1,\eta_0)$ have the remarkable property that the relations between them are very general, being independent of the particular characteristics of the laser beam. In Table 1 we present the variation of η_1 as function of η_0, as it results from (30), for the typical domain of variation of η_0 (see Fig. 9 from [14]), which contains the values corresponding to the cutoff of the HHG spectrum. We also include the corresponding values of $g(\eta_1,\eta_0)$ and of the value of the x component of the kinetic momentum, normalized to $-mca_1$, which corresponds to η_1. In virtue of (18), this component can be written:

$$\frac{p_{x1}}{-mca_1}=\frac{m\gamma v_{x1}}{-mca_1}=\sin\eta_1-\sin\eta_0 \quad (33)$$

We observe that the cutoff of the HHG spectrum, corresponds to $\eta_0=3.4416$, because in this case the value of $p_{x1}/-mca_1$ is maximum.

Table 1. Variations of η_1, $g(\eta_1,\eta_0)$, $p_{x1}/-mca_1$ and z_1/a_0 for typical domain of η_0.

η_0	π	3.1516	3.1916	3.2416	3.3416	3.4416
η_1	3π	9.0787	8.6653	8.3593	7.9238	7.5823
$g(\eta_1,\eta_0)$	π	3.1275	2.9978	2.7693	2.2424	1.7207
$p_{x1}/-mca_1$	0	0.3492	0.7385	0.9748	1.1962	1.2589
z_1/a_0	9.3211	9.2793	8.8945	8.2166	6.6531	5.1053
η_0	3.5416	3.6416	3.8416	4.04159	4.2416	4.7124
η_1	7.2867	7.01871	6.5329	6.0873	5.6649	4.7124
$g(\eta_1,\eta_0)$	1.2574	0.8740	0.3553	0.1065	0.0195	0
$p_{x1}/-mca_1$	1.2328	1.1504	0.8914	0.5887	0.3116	0
z_1/a_0	3.7306	2.5930	1.0543	0.3160	0.0578	0

We consider the particular case when the high harmonics begin to be suppressed, according to data presented in Refs. [19] and [20]. The data corresponding to this domain are as follows: $I=10^{17}W/cm^2$, laser radiation with wavelength $\lambda=0.3\mu m$ and the ionization potential $I_p=15.5$ atomic units, for Ar^{8+}. >From the relations $\omega=2\pi c/\lambda$ and $I=(1/2)\varepsilon_0 cE_M^2$ we obtain $\omega=6.279\times10^{15}rad/s$, $E_M=8.68\times10^{11}V/m$ and $ca_1^2/(2\omega a_0)=2.967$. With the aid of (31) we calculate the values of z_1/a_0, which are shown in Table 1. We observe that z_1/a_0 has a maximum for $\eta_0=\pi$, whose value is $z_1/a_0=9.3211$. The variations of z_1 and p_{x1}, normalized to their maximum values, as functions of η_0 are shown in Fig. 1. From (18) and (21) it follows that, in the vicinity of the point $\eta_0=\pi$, where z_1 is maximum, the total velocity of the electron is negligible, resulting that this domain corresponds to the low frequency part of the HHG spectrum.

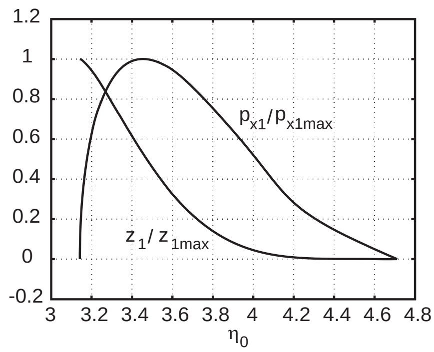

Fig. 1. Variation of $z_1/z_{1\max}$ and $p_{x1}/p_{x1\max}$ with the initial phase η_0

On the other hand, the length of the atomic core, which is equal to the length of the potential barrier width of the nucleus, is given by the relation $l_b=I_p/(eE_M)$ (see Fig. 1 from [15]). Taking into account the previous data, we obtain that $l_b/a_0=9.18$. We observe that this value is very close to the maximum value of z_1/a_0. It follows that, for the regime which has been considered, when the values of p_{x1} are very small, the electron does not enter the atomic core, it does not recombine with the atom, and the HHG process begins to be suppressed. For the same regime, in Refs. [19] and [20] it is shown in a completely different way that the HHG spectrum begins to be suppressed due to the component of the motion of the electron in the z direction, the suppression being greater for the low-frequency part of the HHG spectrum.

We expect that the high energy domain of the HHG spectrum will be suppressed strongly for ultrahigh values of the intensities (higher than $10^{20} W/cm^2$) of the laser beams. Fortunately, at these energies, another mechanism becomes responsible for the generation of very high harmonics, and of attosecond pulses: the relativistic Thomson scattering.

4. Conclusions

In this paper we presented a classical approach of the ionization domain, which is based on the Corkum's hypothesis, in the case of a system composed of an atom in a very intense electromagnetic field. We have shown that, despite the fact that quantum calculations lead to accurate calculations for the ionization rates and for the rates of the generation of high harmonics, the properties of the classical trajectories in the ionization domain offer accurate explanations for numerous effects related to ATI and HHG phenomena.

Acknowledgments

This work was done in the frame of the basic research program of the National Institute for Laser, Plasma and Radiation Physics, entitled "Core Program, 2011"

References

[1] P. Agostini, F. Fabre, G. Mainfray, G. Petite and N. Rahman, Phys. Rev. Lett. **42**, 1127 (1979).
[2] P. B. Corkum, Phys. Rev. Lett. **71**, 1994 (1993).
[3] G. G. Paulus, W. Becker, W. Nicklich, H. Walther, J. Phys. B: At. Mol. Opt. Phys. **27**, L703 (1994).
[4] W. Becker, F. Grasbon, R. Kopold, D. B. Milosevic, G. G. Paulus, H. Walther, Adv. in At. Mol. and Opt. Phys. **48**, 35 (2002).
[5] P. Salieres et al, Science **292**, 902 (2001).
[6] M. Bellini, Phys. Rev. Lett. **81**, 297 (1998).
[7] R. Kopold, D. B. Milosevic and W. Becker, Phys. Rev. Lett. **84**, 3831 (2000).
[8] R. P. Feynman, A. R. Hibbs, Quantum Mechanics and Paths Integrals, McGraw-Hill, New York, 1965.
[9] H. R. Reiss, Phys. Rev. A **22**, 1786 (1980).
[10] F. H. M. Faisal and T. Radozycki, Phys. Rev. A **47**, 4464 (1993).
[11] F. H. M. Faisal and T. Radozycki, Phys. Rev. A **48**, 554 (1993).
[12] M. Lewenstein et al, Phys. Rev. A **49**, 2117 (1994).
[13] C. C. Chirila et al, Phys. Rev. A **66**, 063411 (2002).
[14] M. Protopapas, C. H. Keitel, P. L. Knight, Rep. Prog. Phys. **60**, 389 (1997).
[15] P. Agostini, L. F. DiMauro, Cont. Phys. **49**, 179 (2008).
[16] L. D. Landau, E. M. Lifschitz, The Classical Theory of Fields, Pergamon Press, London, 1959.
[17] E. S. Sarachik S and G. T. Schappert, Phys. Rev. D **1**, 2738 (1970).
[18] R. Uzdin, N. Moiseyev, Phys. Rev. A **81**, 063405 (2010).
[19] V. D. Taranukhin, N. Y. Shubin, J. Opt. Soc. Am. B **19**, 1132 (2002).
[20] V. D. Taranukhin, Laser Phys. **12**, 398 (2002).